**LAB #3: WRITING THE RIGHT WAY?**

**Directions:** You are encouraged to work with a partner on this lab and turn in one report with both names (see end of file for more information).

**Goals for this lab:** In this lab you will see a few different confidence interval procedures and learn how to compare and evaluate their performance. This lab replaces material in Investigations 1.10 and 1.11.

**Names: *>>***

**Study background:** In a previous “initial course survey,” I asked students whether they were right-handed or left-handed, only giving them two options. In this survey, 5 of 30 said they were left-handed.

**Part 1:** Consider a confidence interval for $π$ = probability a 301 student is left-handed. The study observed $\hat{p}$ = 5/30 = 0.1667.

*(exact) Binomial confidence interval:* Here is R and JMP output for an exact “binomial” confidence interval (aka Clopper-Pearson).





*This approach is equivalent to what you did in Investigation 1.6, i.e., “inverting” the binomial distribution to see which values of* $π$ *are not rejected at the 5% level of significance.*

*2SD confidence interval:* Another approach is to find all values of $π$ that are within “2SD” of our observed $\hat{p}$ = 0.1667.

a) But how do we calculate the standard deviation when we don’t know $π$? We could use $\hat{p}$, we could use a hypothesized value of $π$ (e.g., 0.08 say), or we could use 0.50.

1. Using .1667 to calculate the SD. Compute the standard error of $\hat{p}$, SE($\hat{p}$) = $\sqrt{\frac{\hat{p}\left(1-\hat{p}\right)}{n}}$ =*>>*
2. Using .5 to calculate the SD. Compute $\sqrt{\frac{.5\left(1-.5\right)}{n}}$ = *>>*
3. Using .08 to calculate the SD. Compute $\sqrt{\frac{.08\left(1-.08\right)}{n}}$ =*>>*

b) Are these standard deviations very different? *>>*

Which standard deviation is the largest? *>>*

**Key Idea:** You can show mathematically that using 0.50 maximizes this formula. So if you are ever in doubt (or say before you collect any data and you have no historical data, see “sample size determination” on p. 78), you can be conservative and use 0.50.

c) Using the *standard error*, estimate the 95% confidence interval $\hat{p}$ + 2 x SE($\hat{p}$)

 *>>*

The only real downside to this approach is we know 2 is a bit of a lie and we can’t change the confidence level to anything other than 95%.

*One-sample z-interval*: We can invert the normal distribution rather than the binomial distribution. We basically want to invert P($-z\leq \frac{\hat{p}-π}{\sqrt{\frac{π\left(1-π\right)}{n}}}\leq z)=C$ for $π$. Turns out this isn’t quite so simple (could use the quadratic formula). So instead we invert P($-z\leq \frac{\hat{p}-π}{\sqrt{\frac{\hat{p}\left(1-\hat{p}\right)}{n}}}\leq z)=C$ for $π$ and we get $\hat{p}-z\_{C}\sqrt{\frac{\hat{p}\left(1-\hat{p}\right)}{n}}, \hat{p}+z\_{C}\sqrt{\frac{\hat{p}\left(1-\hat{p}\right)}{n}}$, or we could also write this as $\hat{p}\pm z\_{C}\sqrt{\frac{\hat{p}\left(1-\hat{p}\right)}{n}}$, which has that nice form of *statistic* + *margin of error*. *C* here represents our confidence level. **So we determine *zC* to match the confidence level.** We know *zC* should be approximately 2 when the confidence level is 95%. Investigation 1.10 shows you how to find these z values using the standard normal distribution (mean = 0, SD = 1), but these are the most common values:



d) Calculate the 95% one proportion z confidence interval $\hat{p}\pm z\_{C}\sqrt{\frac{\hat{p}\left(1-\hat{p}\right)}{n}}$.

*>>*

e) Use technology (e.g., Theory-based inference applet, iscamonepropztest in R, or JMP, see p. 78) to verify your calculation in (d). Include a screen capture below.

*>>*

f) Use technology to find the 90% one proportion z confidence interval. How do the midpoints and widths compare to (e)? As you expected?

*>>*

You should see that the exact binomial confidence interval (see output above) and the one proportion z-interval from (d) are a bit different. Which is better?

g) Which 95% confidence interval (exact binomial or one proportion z-interval) is wider for this study? (Show your work.)

*>>*

Width is one consideration, but first, we really need to consider whether these really are 95% confidence intervals! What do I mean by that? **A 95% confidence interval *method* should capture the actual value of the parameter of interest 95% of the time in the long run.** What do I mean by that? If I set the process probability $π$ and generate samples from this process, and construct a confidence interval from each sample, what percentage of the resulting intervals succeed in capturing that value of $π$? We can study this using simulation.

**Part 2:** To explore the meaning of confidence and to compare and evaluate these different approaches, let’s pretend we know $π$ = 0.10 and we have a sample size of *n* = 30. Keep in mind that this is an exploration of the method, not an analysis of our sample data.

Open the **Simulating Confidence Intervals** [applet](http://www.rossmanchance.com/applets/ConfSim.html). You will first look at the one proportion z-interval, which this applet calls “Wald” intervals. Specify $π$ = 0.10 and *n* = 30 and say you want **100** intervals. Press **Sample**. If you click on an interval, you will see the sample results for that sample ($\hat{p}$) and you will see where that $\hat{p}$ falls in the distribution of $\hat{p}$ values. If the $\hat{p}$ value happened to be far (more than 1.96SD) from $π$, then the interval will be red, as it fails to capture 0.10 inside the interval. If the $\hat{p}$ value is within 1.96SD of $π$, then the interval is green, as it succeed in capturing 0.10 inside the interval.

Keep pressing Sample until you have a Running Total of 1000 intervals. Screen capture and paste the Running Total output here.

h) Does this method appear to succeed in capturing the value of the parameter 95% of the time?

*>>*

**Key Idea:** Keep in mind that the one proportion z-interval (Wald) is only expected to work when the CLT conditions of large sample size are met.

i) What are the expected number of successes and the expected number of failures for the random process we have created? (Show your work.)

*>>*

In this case, it would be inappropriate to use the one proportion z-interval method as we would not be achieving the claimed 95% confidence level. Using a similar approach, you would see that the binomial interval *is* a 95% confidence interval method. But a downside to the binomial method is the intervals tend to be wider. So then we should ask: is there a confidence interval procedure that achieves 95% confidence interval but tends to be shorter than the binomial intervals? Yes! Thanks to some work done in the 1990s, the following procedure was proposed:

* Add two successes to your sample
* Add two failures to your sample
* Calculate a one-proportion z-interval for this “sample result.”

This is sometimes called the “Plus Four” method.

j) For our *x* = 5 left-handers out of *n* = 30, calculate the observed sample proportion (*X* + 2)/(*n* + 4) = $\tilde{p}$.

*>>*

k) Use this value and new sample size in the previous formula: $\tilde{p}\pm 1.96\sqrt{\frac{\tilde{p}\left(1-\tilde{p}\right)}{n+4}}$.

*>>*

l) How does your Plus Four confidence interval compare to the one proportion z-interval and to the binomial confidence interval from (g)? (Compare midpoints and widths.)

|  |  |  |
| --- | --- | --- |
|  | Midpoint | Width |
| Binomial | *>>*  | *>>* |
| One proportion z-interval | *>>* | *>>* |
| Plus Four interval | *>>* | *>>* |

But to evaluate the *method*, return to the Simulating Confidence Intervals applet and use the third pull-down menu to change from Wald to Plus Four.

(m) Include a screen capture of the Running Total after 1000 intervals.

*>>*

(n) Does this appear to be a 95% confidence interval method? How are you deciding?

*>>*

“Fudging our data” may seem like cheating, but there is actually some nice theory behind this method. The adjustment is not that different in principle from the continuity correction we suggested with p-values. Many statisticians now recommend using this procedure over the one proportion z-interval procedure because it achieves the stated confidence level *for any sample size* and over the binomial interval procedure because the intervals tend to be shorter. In fact, the more general case (something other than 95%) is called the adjusted Wald interval, and it is slowly becoming the normal in many software packages. See p. 85 for a summary of these methods and Stat 418 for more details.

To submit your report:

* Review your answers, both to proofread and to assess your understanding.
* Make sure the screen captures are integrated into the body of Word file.
* Make sure you know where you have saved this file (e.g., the Desktop).
* If submitting together, make sure you have put both names in the Word file.
	+ By putting both names in the file, you are acknowledging that you both contributed substantially to report.
* Have one team member submit the file.
	+ I have tied the assignment to the original groups again but if you decide not to submit together
		- If this is a permanent decision, let me know and I can change the group membership. (You can also let me know if you want me to switch up groups.)
		- If you want to submit multiple files for this assignment, go ahead and it will show me both submissions (but remind me to download both).
		- I have checked a new box that allows me to give different grades to both individuals.

Emailing the assignment to me should be a last resort if uploading is not working.